Micro II Midterm (uc3m-Masters in Economics, March 18, 2025)

Exercise 1. In an economy that extends over two dates, today and tomorrow, there is a single perishable good, consumption, and two consumers, 1 and 2. The state of nature tomorrow can be either *hot* or *cold*. Consumers' preferences for consumption today (x), tomorrow if hot (y), and tomorrow if cold (z) are represented by the utility functions $u_1(x, y, z) = xy$, and $u_2(x, y, z) = xz$, and their endowments are $(\bar{x}_1, \bar{y}_1, \bar{z}_1) = (12, 0, 0)$ and $(\bar{x}_2, \bar{y}_2, \bar{z}_2) = (0, 0, 0)$, respectively. There is a firm owned by Consumer 2 that using x units of consumption today as input produces $4\sqrt{x}$ units of consumption tomorrow if it is hot and $2\sqrt{x}$ if it is cold.

(a) (30 points) Assume that there are contingent markets for all goods. Verify that the prices $(p_x^*, p_y^*, p_z^*) = (1, 3/4, 1/2)$ clear the markets, and calculate the corresponding CE allocation.

(b) (30 points) Now assume that there are no contingent markets but there is a spot market for x and markets for two securities, s and t, operating today. A unit of security s pays tomorrow 1 unit of consumption regardless of the state, while a unit of security t pays tomorrow 2 units of consumption if is hot and nothing if it is cold. In this economy, the firm buys good x to use it as input and sells securities s and t up to its output (i.e., it must honor its contracts), while consumers trade good x as well as securities. Denote the securities prices (in units of the consumption good today per unit of security) by q_s and q_t. Calculate the competitive equilibrium prices of the securities and the corresponding allocation. (Hint. Since the rank of the matrix of security returns equals the number of contingent goods, you may proceed to identify the security prices assuming that, as shown in class for pure exchange economies, the Radner and Arrow-Debreu competitive equilibria of this economy coincide. Then you can verify that the prices you found are indeed the CE prices, and that the allocation is that you found in part (a).)

Exercise 2. *A*nn, *B*ob, and *C*onrad have each a monthly income of 2 thousands euros. They are planning to lease an apartment together and must choose its quality as measured by the monthly rental in thousands of euros, $x \in \mathbb{R}_+$. Their preferences are described by utility functions $u_i(x, y) = y + \alpha_i \ln x$, where y denotes money available to buy other goods and $(\alpha_A, \alpha_B, \alpha_C) = (1, 2/3, 1/3)$.

(a) (10 points) Identify the set Pareto optimal allocations.

(b) (10 points) Calculate the Lindahl equilibrium.

(c) (10 points) Calculate the monthly rental resulting from voluntary contributions. (Keep in mind that contributions must be non-negative.)

(d) (10 points) What will be the monthly rental if they share equally the cost and adopt the median of the proposals made by Ann, Bob and Conrad?

Solutions

1(a). I normalize $p_x = 1$, and proceed to identify the CE prices. The firm chooses its input x solving the problem

$$\max_{x \geq 0} \ (4p_y + 2p_z)\sqrt{x} - x.$$

The first order condition for a solution to this problem is

$$(4p_y + 2p_z)\frac{1}{2\sqrt{x}} - 1 = 0.$$

Hence the firm's input demand of and supply of goods are

$$x_f(p_y, p_z) = (2p_y + p_z)^2, \ y_f(p_y, p_z) = 4(2p_y + p_z), \ z_f(p_y, p_z) = 2(2p_y + p_z),$$

and its profit is

$$\pi(p_y, p_z) = 4p_y(2p_y + p_z) + 2p_z(2p_y + p_z) - (2p_y + p_z)^2 = (2p_y + p_z)^2.$$

Consumer 1's problem is

$$\max_{(x,y,z) \in \mathbb{R}_+} xy, \ \textit{subject to: } x + p_y y + p_z z = 12.$$

Hence, her good demands are

$$x_1(p_y, p_z) = 6, \ y_1(p_y, p_z) = \frac{6}{p_y}, \ z_1(p_y, p_z) = 0.$$

Consumer 2's problem is

$$\max_{(x,y,z) \in \mathbb{R}_+} xz, \ \textit{subject to: } x + p_y y + p_z z = (2p_y + p_z)^2.$$

Hence, her good demands are

$$x_2(p_y, p_z) = \frac{(2p_y + p_z)^2}{2}, \ y_2(p_y, p_z) = 0, \ z_2(p_y, p_z) = \frac{(2p_y + p_z)^2}{2p_z}.$$

Market Clearing requires

$$(\text{Good } y) \qquad \frac{6}{p_y} = 4(2p_y + p_z); \ (\text{Good } z) \qquad \frac{(2p_y + p_z)^2}{2p_z} = 2(2p_y + p_z)$$

Solving this system of equations we get

$$(p_y^*, p_z^*) = (\frac{3}{4}, \frac{1}{2}).$$

Thus, in the CE the firm's production activity and profit are

$$\left(x_f^*, y_f^*, z_f^*\right) = (4, 8, 4), \ \textit{and} \ \pi^* = 4,$$

and consumers' consumptions are

$$[(x_1^*, y_1^*, z_1^*), (x_2^*, y_2^*, z_2^*)] = [(6, 8, 0), (2, 0, 4)].$$

1(b). The budget constrains of Consumer i in this economy are

$$\begin{aligned}
&(1)\ x_i + q_s s_i + q_t t_i = I_i \\
&(2)\ y_i = s_i + 2t_i \\
&(3)\ z_i = s_i,
\end{aligned}$$

where $I_1 = 4$ *and* $I_2 = \pi(q_s, q_t)$. *Using equations (2) and (3) to calculate* s_i and t_i *we may write the consolidated budget constraint involving consumption goods as*

$$x_i + \frac{q_t}{2} y_i + \left(q_s - \frac{q_t}{2}\right) z_i = I_i.$$

Following on the suggestion given in the hint, in equilibrium the effective prices of goods y and z must coincide with those of part (a), i.e.,

$$\begin{aligned}
\frac{q_t^*}{2} &= p_y^* = \frac{3}{4} \\
\left(q_s^* - \frac{q_t^*}{2}\right) &= p_z^* = \frac{1}{2}.
\end{aligned}$$

Thus,

$$(q_s^*, q_t^*) = (\frac{3}{2}, \frac{5}{4}).$$

Let us verify that these prices would lead the firm to choose the same production plan as in part (a), The firm's problem is

$$\begin{aligned}
&\max_{x,s,t}\ q_s^* s + q_t^* t - x. \\
s.t.\ :\quad &4\sqrt{x} = s + 2t,\ 2\sqrt{x} = s.
\end{aligned}$$

Using the constraints to substitute $s = 2\sqrt{x}$ *and* $t = \sqrt{x}$ *and the security prices calculated above we write the problem as*

$$\max_{x \geq 0}\ \frac{5}{4}\left(2\sqrt{x}\right) + \frac{3}{2}\left(\sqrt{x}\right) - x = 4\sqrt{x} - x.$$

The solution to this problem and the firm's profit are the same production activity as in the CE identified in part (a), i.e., $x^* = 4$ *and* $\pi^* = 4$.

Obviously, since consumes 1 and 2 have the same income and buy goods at the same prices as in part (a), their demands are the same. Hence, these security prices are indeed a CE, and generate the same allocation as in part (a).

2(a). A Pareto optimal level of public good must satisfy equation

$$\sum_{i\in\{A,B,C\}} MRS_i(x,y) = \frac{2}{x} = 1$$

Thus, any allocation (x, y_A, y_B, y_C) *such that* $x = 2$ *and* $y_A + y_B + y_B = 6 - 2$ *is Pareto optimal.*

2(b). In a Lindahl equilibrium the system of personalized prices must be such that for $i \in \{A, B, C\}$

$$MRS_i(x,y) = \frac{\alpha_i}{x} = p_i.$$

Moreover, since Lindahl equilibria are Pareto optimal, the monthly rental must be $x = 2$*. Hence* $p_A = 1/2$, $p_B = 1/3$, $p_C = 1/6$*. Incomes after paying the monthly rental according to Lindahl prices are*

$$(y_A^L, y_B^L, y_C^L) = (2 - 2p_A, 2 - 2p_B, 2 - 2p_C) = (1, \frac{4}{3}, \frac{5}{3}).$$

2(c). Individual $i \in \{A, B, C\}$ *decides her contribution* $z_i \in R_+$ *by solving the problem*

$$\max_{z_i\in\mathbb{R}_+} y + \alpha_i \ln(z_i + Z_{-i}), \text{ subject to: } y + z_i = 2.$$

where $Z_{-i} = \sum_{j\neq i} z_j$*. This problem is equivalent to*

$$\max_{z_i\in\mathbb{R}_+}(2 - z_i) + \alpha_i \ln(z_i + Z_{-i}).$$

The FOC for an interior solution to this problem is

$$-1 + \frac{\alpha_i}{z_i + Z_{-i}} = 0 \Leftrightarrow z_i = r_i(Z_{-i}) = \alpha_i - Z_{-i}.$$

If $r_i(Z_{-i}) < 0$*, then* $z_i = 0$*, since a negative contribution is not feasible. That is*

$$z_i^* = \max\{\alpha_i - Z_{-i}, 0\}.$$

Hence $z_A^* \geq \alpha_A - (z_B^* + z_C^*)$*, for otherwise Ann would increase her contribution. Then* $z_B^* = z_C^* = 0$*, and therefore* $z_A^* = 1$*. The resulting allocation is*

$$(x^V, y_A^V, y_B^V, y_C^V) = (1, 1, 2, 2).$$

2(d). As discuss in class, it is a dominant strategy for each individual to propose her optimal monthly rental, that is, individual $i \in \{A, B, C\}$ *decides her proposal* $v_i \in R_+$ *by solving the problem*

$$\max_{v\in\mathbb{R}_+} \left(2 - \frac{v}{3}\right) + \alpha_i \ln v.$$

The FOC for a solution to this problem is

$$-\frac{1}{3} + \frac{\alpha_i}{v} = 0 \Leftrightarrow v_i^* = 3\alpha_i.$$

Hence votes are $(v_A^*, v_B^*, v_C^*) = (3, 2, 1)$*, and therefore* $x_M^* = 2$*, and* $(y_A^M, y_B^M, y_C^M) = (4/3, 4/3, 4/3)$*.*